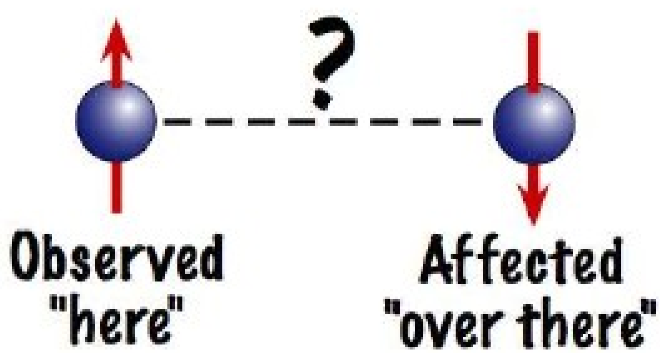
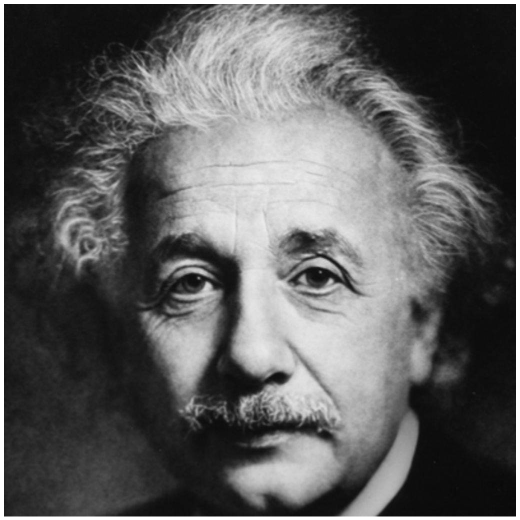
Entanglement

Quantum entanglement is a physical phenomenon which occurs when pairs or groups of particles are generated, interact, or share spatial proximity in ways such that the quantum state of each particle cannot be described independently of the state of the other(s), even when the particles are separated by a large distance



https://www.uh.edu/engines/epi2727.htm

Entanglement



Albert **E**instein Boris **P**odolsky Nathan **R**osen

EPR

Entanglement

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

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(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

The New York Times, Same Year (1935)

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

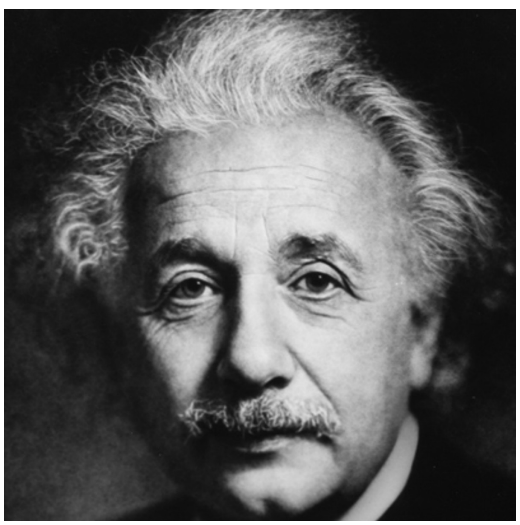
SEE FULLER ONE POSSIBLE

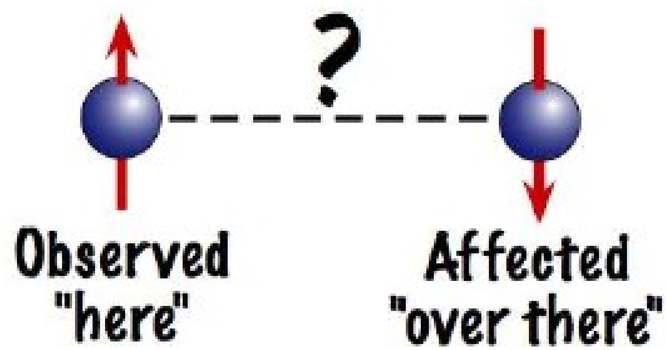
Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Entanglement

"Spukhafte Fernwirkung"

"spooky action at a distance"





Entanglement

- Everything was fine for EPR until 1964

John Stewart Bell (28 June 1928 – 1 October 1990) was a Northern Irish physicist

He proved that the principle of locality was mathematically inconsistent with the predictions of quantum theory.

Entanglement

- We can summarize the conventional or classical view as *local realism* and the theories that are based on this philosophy as *local realistic theories*
  - Locality. Measurement of particle A in no way disturbs the state of spatially separated particle B.
  - Realism. The values of measurable properties of each particle are objectively real. They have definite values prior to measurement and regardless of whether or not they are observed.

Entanglement

David Joseph Bohm (December 20, 1917 – October 27, 1992) was an American scientist who has been described as one of the most significant theoretical physicists of the 20th century and who contributed unorthodox ideas to quantum theory, neuropsychology and the philosophy of mind.

1952 → Thought Experiment

He considered a spin-0 particle that decays into two spin-1/2 particles.

A total spin-0 state, which consists of a system of two spin-1/2 particles, can be described in terms of the computational basis with the *singlet* state

$$|\psi\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

Entanglement

- We can make a measurement of Z on the first particle, while leaving the second particle alone using the operator $Z \otimes I$

- if we measure 0 for the first particle, the state of the second particle *must be* $|1>$

- On the other hand, if we measure 1 for the first particle, the state of the second particle must be $|0>$

Entanglement

- Let’s see what happens if we rewrite this state in terms of the eigenvectors of the *X* operator, the $|\pm\rangle$ states
- Recall that

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}, \quad |1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

Entanglement

$$|0\rangle|1\rangle = \left(\frac{|+\rangle + |-\rangle}{\sqrt{2}}\right)\left(\frac{|+\rangle - |-\rangle}{\sqrt{2}}\right) = \frac{1}{2}(|++\rangle + |-+\rangle - |+-\rangle - |--\rangle)$$

$$|1\rangle|0\rangle = \left(\frac{|+\rangle - |-\rangle}{\sqrt{2}}\right)\left(\frac{|+\rangle + |-\rangle}{\sqrt{2}}\right) = \frac{1}{2}(|++\rangle - |-+\rangle + |+-\rangle - |--\rangle)$$

Hence $$\psi\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{1}{2}(|++\rangle + |-+\rangle - |+-\rangle - |--\rangle - |++\rangle + |-+\rangle - |+-\rangle + |--\rangle)$$

$$= \frac{1}{\sqrt{2}}\frac{1}{2}(2|-+\rangle - 2|+-\rangle)$$

$$= -\frac{(|+-\rangle - |-+\rangle)}{\sqrt{2}}$$

Entanglement

- If we measure the *X* operator for the first particle and obtain the result +, the second particle must be in the state $|->$, and vice versa.

- The fact that the states are also correlated in the *x* direction indicates that the particles cannot be in definite states prior to measurement and that the states are part of a larger composite system.

Entanglement

- Recall that

$$[X, Y] = 2iZ, \quad [Y, Z] = 2iX, \quad [Z, X] = 2iY$$

- The fact that $[Z, X] = 2iY \neq 0$ reminds us that the state of the second particle can't be an eigenstate of both the Z operator and the X operator.

Entanglement

If we only assume that conservation of angular momentum holds and say nothing about quantum mechanics, then the measurement results with respect to the Pauli operators *Z* , *X* will be found as listed in the following Table

Conservation of momentum by particles Alice and Bob

Alice z Direction	Alice x Direction	Bob z Direction	Bob x Direction
$+1$	$+1$	-1	-1
$+1$	-1	-1	$+1$
-1	$+1$	$+1$	-1
-1	-1	$+1$	$+1$

The results agree with quantum mechanics.

Entanglement-Bell’s Theorem

In 1964, John S. Bell by considering spin measurements along three nonorthogonal directions, which are conventionally labeled a, b, and c, derived an inequality that is satisfied by local realistic theories but violated by quantum mechanics. This inequality is experimentally testable, and to date all experimental evidence has come down in favor of quantum mechanics.

Bell's theorem is a "no-go theorem"

Entanglement-Bell's Theorem

We begin our thought experiment by imagining that an ensemble or large number of systems have been prepared so that Alice and Bob can measure spin along the three directions **a, b,** and **c**.

Measurements for particles Alice and Bob

	Alice			Bob		
Population	a	b	c	a	b	c
N_1	+	+	+	−	−	−
N_2	+	+	−	−	−	+
N_3	+	−	+	−	+	−
N_4	+	−	−	−	+	+
N_5	−	+	+	+	−	−
N_6	−	+	−	+	−	+
N_7	−	−	+	+	+	−
N_8	−	−	−	+	+	+

$$N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8$$

Entanglement-Bell's Theorem

All possible measurements for particles Alice and Bob considering population N_4

Direction			
Alice	Bob	Alice	Bob
a	**b**	+1	+1
a	**c**	+1	+1
b	**a**	−1	−1
b	**c**	−1	+1
c	**a**	−1	−1
c	**b**	−1	+1

Entanglement-Bell's Theorem

Some Basic Math

$$N_i \geq 0 \text{ and } N = \sum_{i=1}^{8} N_i$$

if *x*, *y*, *z* are real numbers such that *x* ≥ 0, *y* ≥ 0, *z* ≥ 0, then *x* + *y* ≤ *x* + *y* + *z*. So it must be true that

N3* + *N4* ≤ *(N3* + *N4)* + *(N2* + *N7)* = *(N2* + *N4)* + *(N3* + *N7)

We can divide by *N*, the total number in the ensemble,

$$\frac{N_3 + N_4}{N} \leq \frac{(N_2 + N_4)}{N} + \frac{(N_3 + N_7)}{N}$$

Entanglement-Bell's Theorem

We see that *N* 3 and *N* 4 both have Alice measuring +1 along **a** and Bob measuring+1 along **b**. Hence

$$\frac{N_3 + N_4}{N} = \Pr(+ \mathbf{a}; +\mathbf{b})$$

Similarly

$$\frac{N_2 + N_4}{N} = \Pr(+\mathbf{a}; +\mathbf{c})$$

$$\frac{N_3 + N_7}{N} = \Pr(+\mathbf{c}; +\mathbf{b})$$

Bell's inequality

$$\Pr(+\mathbf{a}; +\mathbf{b}) \leq \Pr(+\mathbf{a}; +\mathbf{c}) + \Pr(+\mathbf{c}; +\mathbf{b})$$

Derived under the assumption of *Local Realism*.

Entanglement-Bell's Theorem

- Local realistic theories *satisfy* or obey Bell's inequality.
- As we'll see, quantum mechanics *does not*.
- To see what quantum mechanics says about the situation, we need to consider a qubit oriented in an arbitrary direction.

Entanglement-Bell's Theorem

Consider a unit vector

$$\vec{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

The eigenvectors of $\vec{\sigma}\cdot\vec{n}$ are

$$|+_n\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$
$$|-_n\rangle = \cos\frac{\theta}{2}|0\rangle - e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Entanglement-Bell's Theorem

If the system is in the state |+$_n$>, *then we have*

$$\langle 0|+_n\rangle = \cos\frac{\theta}{2}$$

Therefore the probability that measurement finds |0> given that the system is in the state |+$_n$> *is*

$$|\langle 0|+_n\rangle|^2 = \cos^2\left(\frac{\theta}{2}\right)$$

Similarly the probability to find |1> given that the system is in the state |−$_n$ > *is*

$$|\langle 1|+_n\rangle|^2 = \sin^2\left(\frac{\theta}{2}\right)$$

Entanglement-Bell's Theorem

Now that we see how to relate some axis *to the z axis, we can write down* relations for arbitrary axes a, b, and c.

We define the angles between each of these axes as ϑ_{ab}, ϑ_{cb}, *and ϑ_{ac}, respectively.*

We prepare the system in the singlet state. The form of the state is invariant under rotations, so we consider it along the a axis:

$$|\psi\rangle = \frac{|+_a\rangle|-_a\rangle - |-_a\rangle|+_a\rangle}{\sqrt{2}}$$

Entanglement-Bell's Theorem

Then we can calculate Pr(+a;+c) by looking at the inner product

$$\langle +_a +_c | \psi \rangle = \frac{\langle +_a | +_a \rangle \langle +_c | -_a \rangle - \langle +_a | -_a \rangle \langle +_c | +_a \rangle}{\sqrt{2}}$$

$$= \frac{\langle +_c | -_a \rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \sin \frac{\theta_{ac}}{2}$$

Hence the probability is

$$\Pr(+\mathbf{a}; +\mathbf{c}) = \left| \frac{\langle +_c | -_a \rangle}{\sqrt{2}} \right|^2 = \frac{1}{2} \sin^2 \left(\frac{\theta_{ac}}{2} \right)$$

Entanglement-Bell’s Theorem

Similarly we find that

$$\Pr(+\mathbf{a};+\mathbf{b})=\frac{1}{2}\sin^2\left(\frac{\theta_{ab}}{2}\right),\quad \Pr(+\mathbf{c};+\mathbf{b})=\frac{1}{2}\sin^2\left(\frac{\theta_{cb}}{2}\right)$$

This means that we can write Bell’s inequality as

$$\sin^2\left(\frac{\theta_{ab}}{2}\right)\le\sin^2\left(\frac{\theta_{ac}}{2}\right)+\sin^2\left(\frac{\theta_{cb}}{2}\right)$$

Entanglement-Bell's Theorem

We take a, b, and c to lie in a plane with c bisecting the angle θ_{ab}. We then obtain the simplification $\theta_{ac} = \theta_{cb} = \theta$, $\theta_{ab} = 2\theta$, and Bell's inequality becomes $\sin^2(\theta) \le 2 \sin^2(\theta/2)$. Bell's inequality is violated when

$$0 < \theta < \frac{\pi}{2}$$

Suppose that Alice and Bob design their system such that $\theta = \pi/3$. Bell's inequality is then the nonsensical statement that $0.75 \le 0.5$. Therefore quantum mechanics clearly predicts a violation of Bell's inequality, which was derived under the assumption of local realism.