



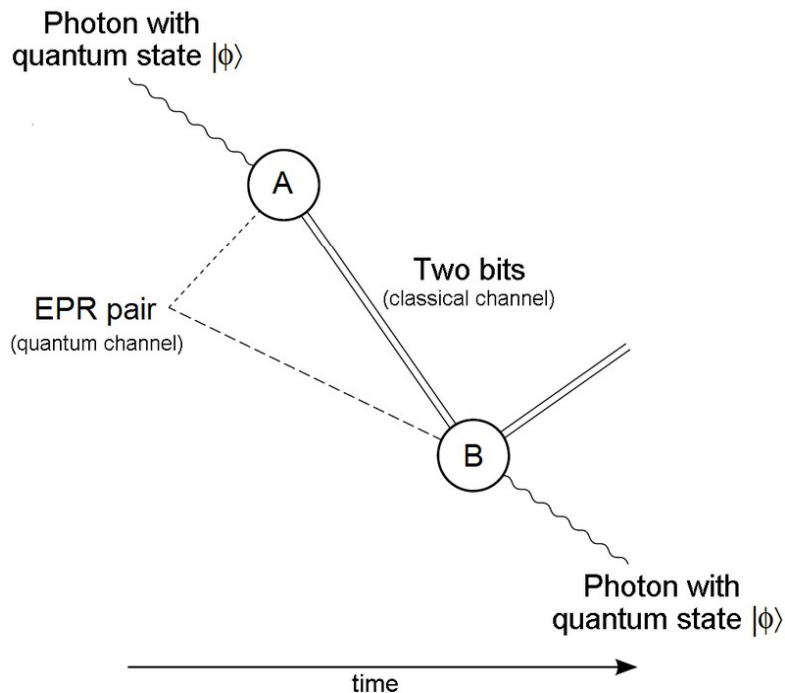
Quantum Teleportation over a Noisy Channel

Hossein AGHABABA

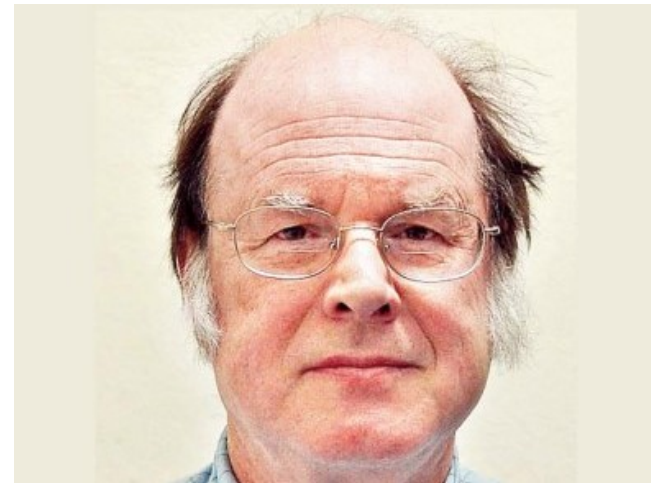
Invited Professor-Sorbonne University

Quantum Teleportation

A process in which quantum information (e.g. the exact state of an atom or photon) can be transmitted from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location.



The term was coined by physicist Charles Bennett [1].



[1] Bennett, C.H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels. Phys. Rev. Lett. **70(13)**, 1895 (1993)

Quantum Teleportation-**Experimental demonstration**

- In 1997, the first prosperous experimental demonstration of quantum teleportation was proposed by Bouwmeester [1] based on Benett et al. protocol.

Dirk Bouwmeester



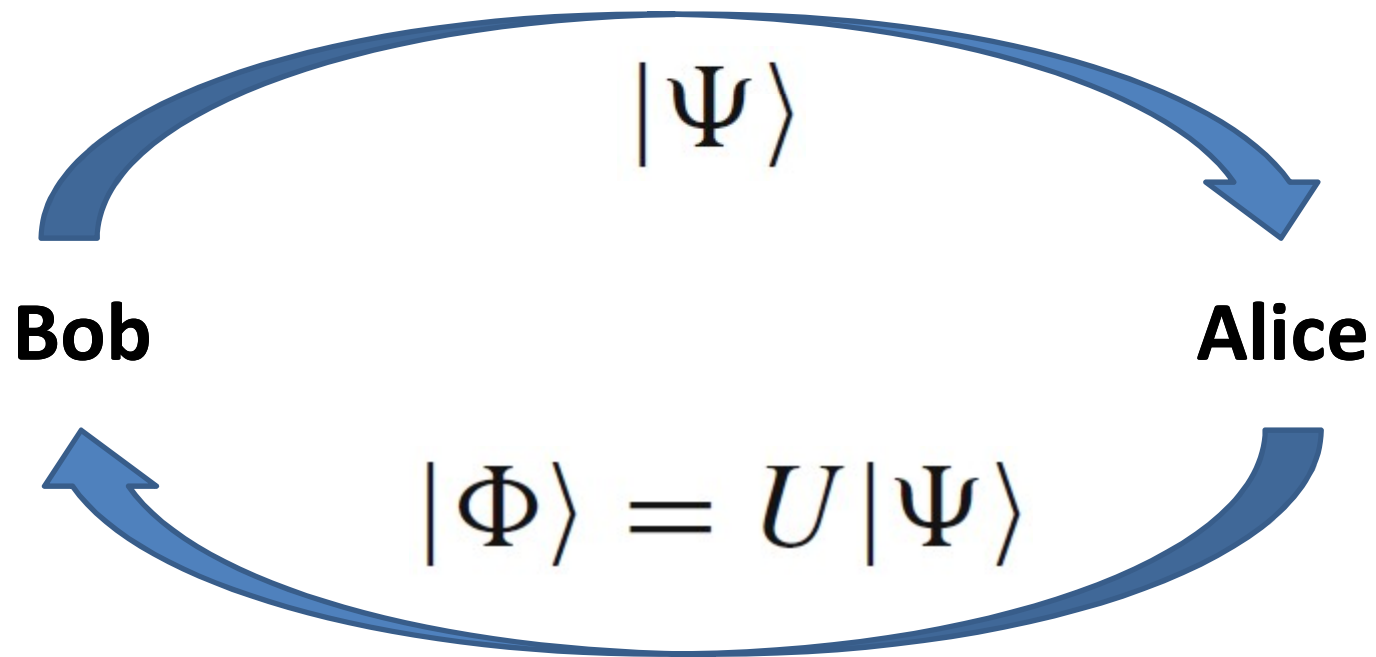
[1] Bouwmeester, D., Pan, J.-W., Mattle, K., Eibl, M., Weinfurter, H., Zeilinger, A.: Experimental quantum teleportation. *Nature* **390(6660)**, 575–579 (1997)

Quantum Teleportation-Types

- quantum controlled teleportation (QCT)
- bidirectional quantum teleportation (BQT)
- bidirectional quantum controlled teleportation (BQCT)
- multiparty-controlled teleportation (MCT)

Bidirectional Quantum Teleportation (BQT)

- In a BQT scheme, Bob can teleport a quantum state to Alice, and then Alice applies a unitary operator U on *that and teleports back the state to Bob*.



Bidirectional Quantum Teleportation (BQT)

- BQT can be used to implement a quantum remote control or a nonlocal quantum gate [1,2].
- The bidirectional state teleportation is an essential part of protocol design in cryptographic applications such as quantum key distribution (QKD) and quantum secret sharing (QSS) [3,4].

[1] Huelga, S.F., Plenio, M.B., Vaccaro, J.A.: Remote control of restricted sets of operations: teleportation of angles. *Phys. Rev. A* **65(4)**, 042316 (2002)

[2] Thapliyal, K., Pathak, A.: Applications of quantum cryptographic switch: various tasks related to controlled quantum communication can be performed using bell states and permutation of particles. *Quantum Inf. Process.* **14(7)**, 2599–2616 (2015)

[3] Deng, F.-G., Long, G.L.: Bidirectional quantum key distribution protocol with practical faint laser pulses. *Phys. Rev. A* **70(1)**, 012311 (2004)

[4] Deng, F.-G., Zhou, H.-Y., Long, G.L.: Bidirectional quantum secret sharing and secret splitting with polarized single photons. *Phys. Lett. A* **337(4–6)**, 329–334 (2005)

Our proposed protocol

- Goal: Teleportation of arbitrary $2n$ *qubits* between Alice and Bob simultaneously [1]

(Published in Quantum Information Processing Journal in October 2019)

[1] Sadeghi-Zadeh, M.S., Houshmand, M., Aghababa, H. et al. Quantum Inf Process (2019) 18: 353. <https://doi.org/10.1007/s11128-019-2456-6>

Our proposed protocol

- The states of Alice's and Bob's qubits are given in the following equations:

$$|\Phi\rangle_{A_1 A_2 \dots A_n} = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle,$$

$$|\Phi\rangle_{B_1 B_2 \dots B_n} = \sum_{j=0}^{2^n-1} \beta_j |j\rangle,$$

Where $\sum_{i=1}^n |\alpha_i|^2 = \sum_{i=1}^n |\beta_i|^2 = 1$

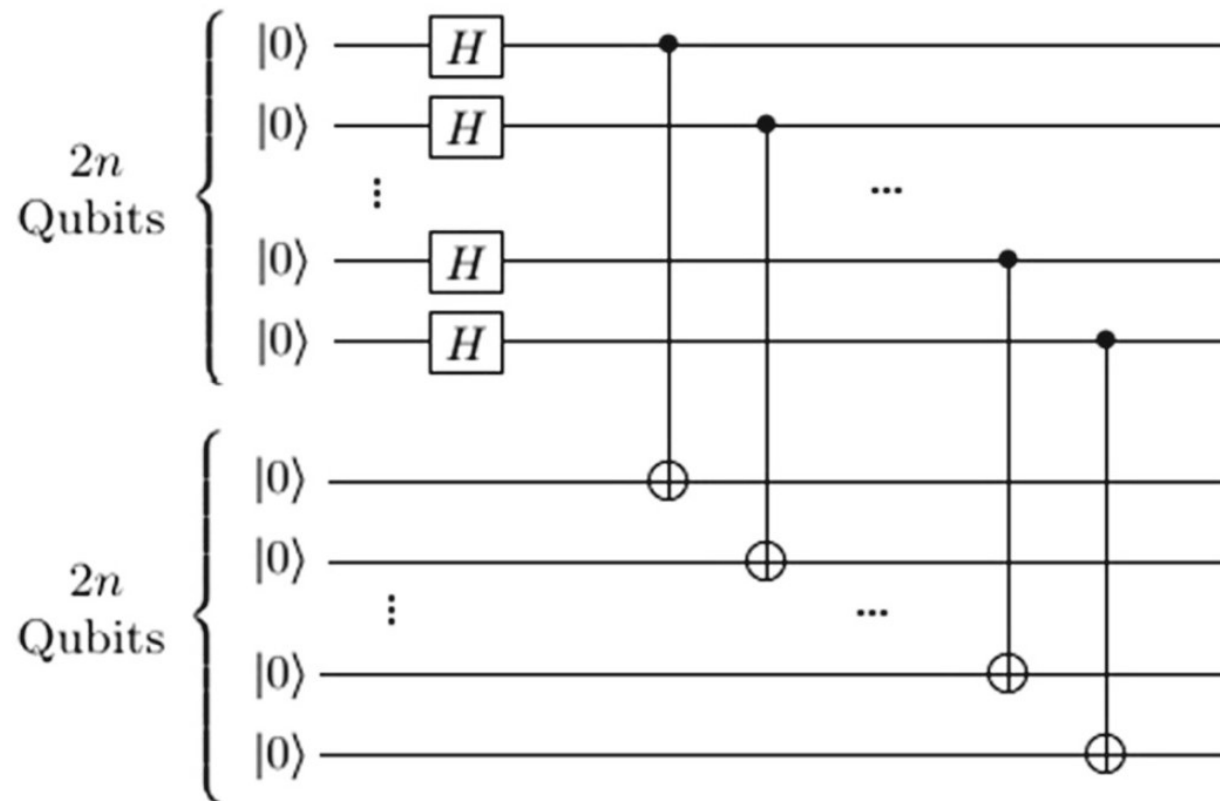
i and j are decimal values of $(i_1, \dots, i_n)_2$ and $(j_1, \dots, j_n)_2$

Our proposed protocol

- The quantum channel of the proposed protocol is given in the following equation:

$$|G\rangle_{a_1 \dots a_n b_1 \dots b_n b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n}} = \frac{1}{2^{2n}} \sum_{l,k=0}^{2^n-1} |l\rangle |k\rangle |l\rangle |k\rangle.$$

Our proposed protocol



Architecture of the proposed $4n$ -channel

Our proposed protocol

- Alice (Bob) receives the qubits $a_1 \dots a_{2n}(b_1 \dots b_{2n})$ from $|G\rangle$.
- The state of the whole $6n$ -qubit quantum system is given in the following equation:

$$|\Phi\rangle_{a_1 \dots a_n b_1 \dots b_n b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n} A_1 \dots A_n B_1 \dots B_n} = |G\rangle \otimes |\Phi\rangle_{A_1 \dots A_n} \otimes |\Phi\rangle_{B_1 \dots B_n}.$$

Our proposed protocol

- The proposed protocol in five steps:

Step 1: Alice (Bob) performs CNOT operators where $A_1 A_2 \dots A_n (B_1 B_2 \dots B_n)$ are the control qubits and $a_1 a_2 \dots a_n (b_1 b_2 \dots b_n)$ are the target qubits, respectively. The state of the whole system after performing CNOT operators is given as follows:

$$\begin{aligned}
 & |\Phi\rangle_{A_1 \dots A_n B_1 \dots B_n a_1 \dots a_n b_1 \dots b_n b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n}} \\
 &= \frac{1}{2^{2n}} \sum_{i,j,k,l=0}^{2^n-1} \alpha_i \beta_j |i\rangle |j\rangle |l \oplus i\rangle |k \oplus j\rangle |l\rangle |k\rangle.
 \end{aligned}$$

Our proposed protocol

Step 2: In this step, Alice and Bob measure qubits $a_1 a_2 \dots a_n$ and $b_1 b_2 \dots b_n$ in Z basis, respectively. The results of this step are shown in Table 1.

Table 1 Measurement results of Step 2 and the corresponding applied operators in Step 3

Alice's results	Bob's results	Collapsed states of qubits $A_1 \dots A_n B_1 \dots B_n b_{n+1} \dots b_{2n} a_{n+1} a_{2n}$	Operators by Bob applied on $b_{n+1} \dots b_{2n}$	Operators by Alice applied on $a_{n+1} \dots a_{2n}$
0...0	0...0	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$I^{\otimes n}$	$I^{\otimes n}$
0...0	0...01	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle i_1 \dots i_n\rangle j_1 \dots j_{n-1} \bar{j}_n\rangle$	$I^{\otimes n}$	$I^{\otimes(n-1)} X$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
0...0	1...11	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle i_1 \dots i_n\rangle \bar{j}_1 \dots \bar{j}_{n-1} \bar{j}_n\rangle$	$I^{\otimes n}$	$X^{\otimes n}$
0...1	0...00	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle i_1 \dots \bar{i}_n\rangle j_1 \dots j_{n-1} j_n\rangle$	$I^{\otimes(n-1)} X$	$I^{\otimes(n)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1...1	1...10	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle \bar{i}_1 \dots \bar{i}_n\rangle \bar{j}_1 \dots \bar{j}_{n-1} j_n\rangle$	$X^{\otimes n}$	$X^{\otimes(n-1)} I$
1...1	1...1	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle \bar{i}_1 \dots \bar{i}_n\rangle \bar{j}_1 \dots \bar{j}_{n-1} \bar{j}_n\rangle$	$X^{\otimes n}$	$X^{\otimes n}$

Our proposed protocol

Step 3: Alice and Bob apply X and I operators as shown in the fourth and fifth columns of Table 1, respectively. All states result in the following equation:

$$|\phi\rangle_{A_1 \dots A_n B_1 \dots B_n b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n}} = \frac{1}{2^{2n}} \sum_{i,j=0}^{2^n-1} \alpha_i \beta_j |i\rangle |j\rangle |i\rangle |j\rangle.$$

Our proposed protocol

Step 4: Alice and Bob perform X-basis measurement on $A_1 A_2 \dots A_n$ and $B_1 B_2 \dots B_n$, respectively. The measurement results and the corresponding states after the measurements are shown in Table 2.

Table 2 Measurement results of Step 4 and the corresponding applied operators in Step 5

Alice's results	Bob's results	Collapsed states of qubits $b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n}$	Operators by Bob applied on $b_{n+1} \dots b_{2n}$	Operators by Alice applied on $a_{n+1} \dots a_{2n}$
$+\dots +$	$+\dots +$	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$I^{\otimes n}$	$I^{\otimes n}$
$+\dots +$	$+\dots + -$	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} (-1)^{j_n} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$I^{\otimes n}$	$I^{\otimes(n-1)} Z$
...	...	...	...	...
$+\dots +$	$-\dots -$	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} (-1)^{\sum_{h=1}^n j_h} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$I^{\otimes n}$	$Z^{\otimes n}$
$+\dots + -$	$+\dots +$	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} (-1)^{i_n} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$I^{\otimes(n-1)} Z$	$I^{\otimes n}$
...	...	...	...	...
$-\dots -$	$-\dots - +$	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} (-1)^{\sum_{h=1}^n i_h + \sum_{h=1}^{n-1} j_h} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$Z^{\otimes n}$	$Z^{\otimes(n-1)} I$
$-\dots -$	$-\dots -$	$\sum_{i_1, \dots, i_n, j_1, \dots, j_n \in \{0,1\}} (-1)^{\sum_{h=1}^n (i_h + j_h)} \alpha_i \beta_j i_1 \dots i_n\rangle j_1 \dots j_n\rangle$	$Z^{\otimes n}$	$Z^{\otimes n}$

Our proposed protocol

Step 5: The last step of the proposed method focuses on correcting the collapsed states by applying Z operators as detailed in Table 2. After applying the mentioned Z operators, the state of whole system is as follows:

$$|\phi\rangle_{b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n}} = \sum_{i,j=0}^{2^n-1} \alpha_i \beta_j |i\rangle |j\rangle.$$

Our proposed protocol

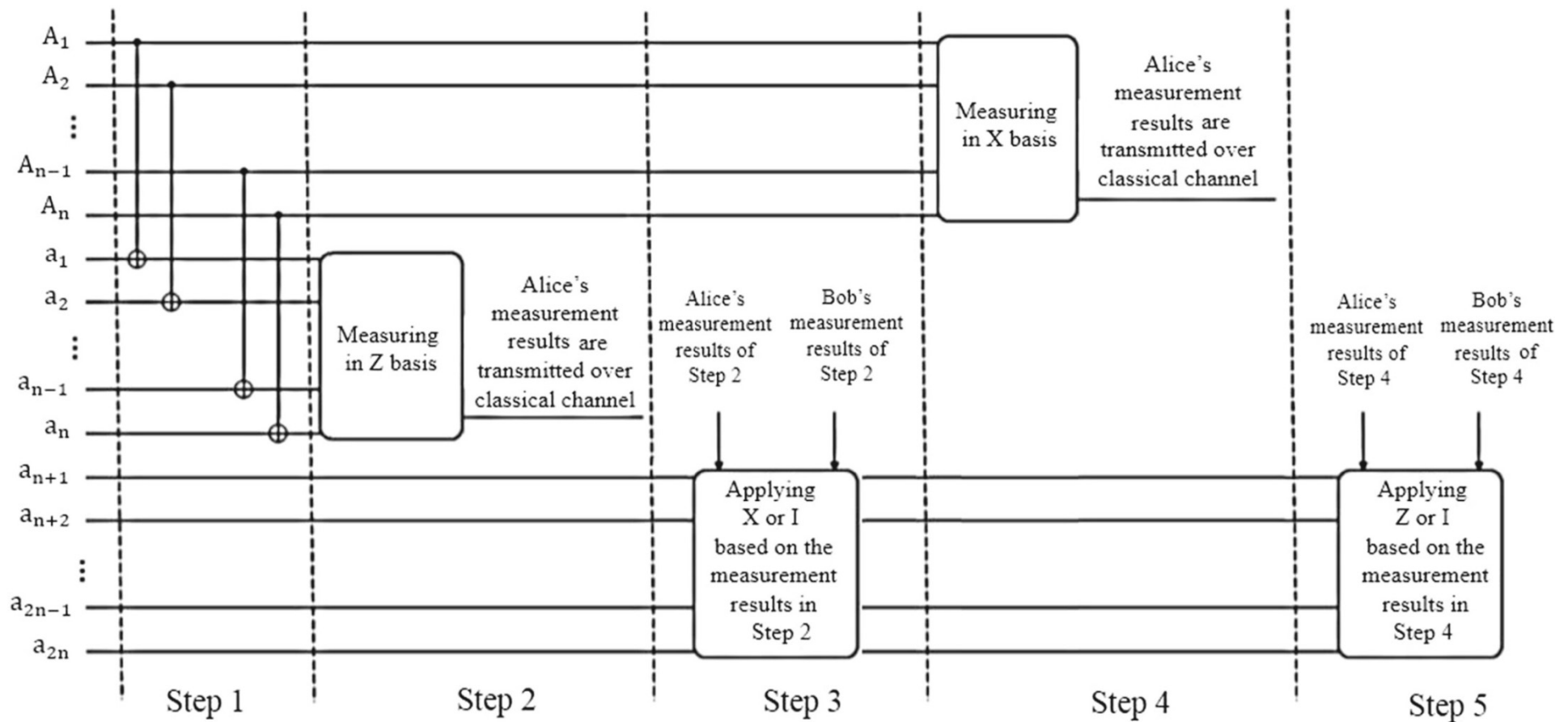
- Alice's and Bob's final states are shown in the following equations:

$$|\Phi\rangle_{a_{n+1}\dots a_{2n}} = \sum_{i=0}^{2^n-1} \beta_i |i\rangle,$$

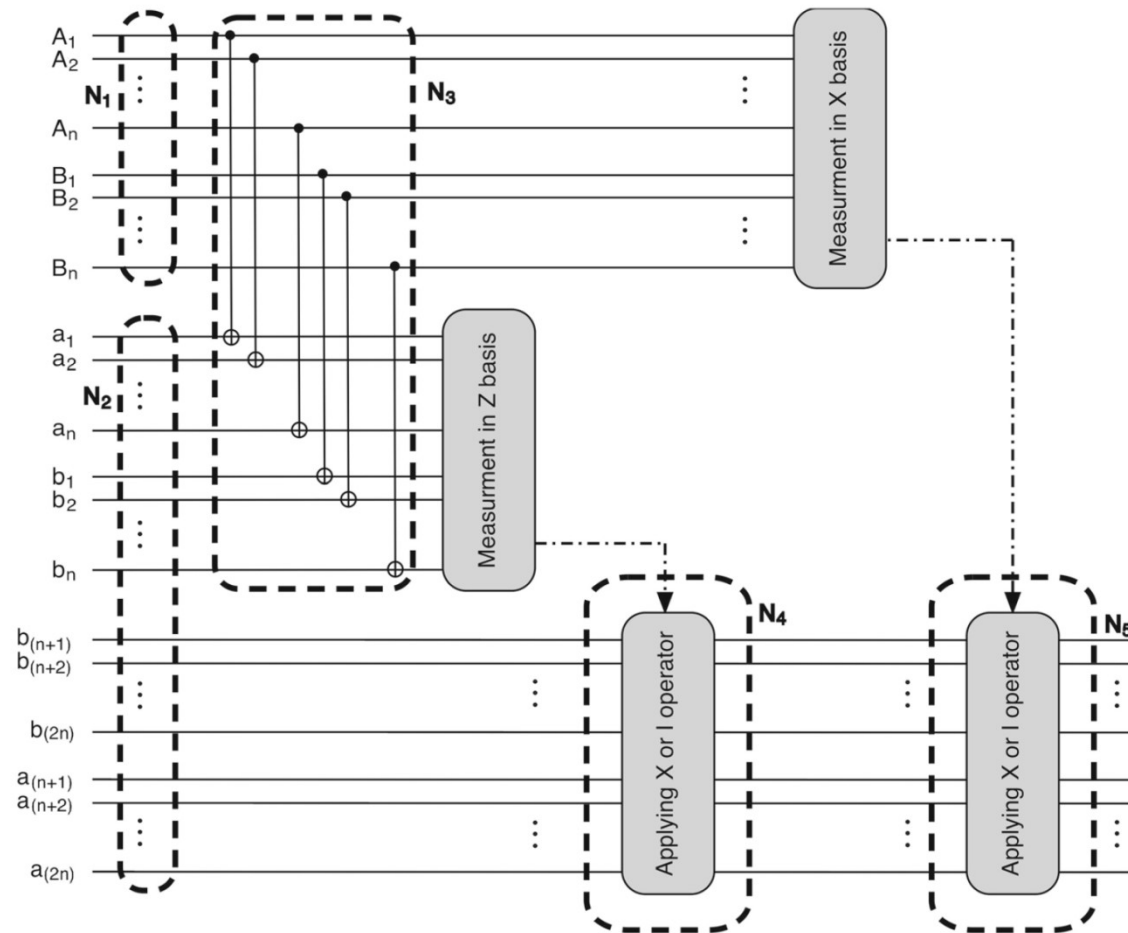
$$|\Phi\rangle_{b_{n+1}\dots b_{2n}} = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle.$$

Our proposed protocol

Architecture of the proposed protocol on Alice's side



Noise Analysis



All parts which can be affected by noise in our quantum teleportation system

Noise Analysis

- In case *N1*, *our unknown input state loses its coherence* before it is teleported
- In case *N2*, *the entangled qubits make quantum channel* noisy while being shared and kept by Alice and Bob
- In cases *N3, N4 and N5*, *when Alice and Bob perform the operations*, the impact of noise may become notable

Noise Analysis

- In this section, we investigate the effect of noise on input qubits ($N1$) and channel qubits ($N2$), for some small case $n = 1$ and $n = 2$, considering the evolution of a noisy system through a Markov evolution model

Noise Analysis

- In our open quantum system after tracing out the environment degree of freedom, the dynamics of the density matrix of all our input and channel qubits can be described by a master equation in the Lindblad form [1],[2],[3]

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_s, \rho] + \sum_{i,\alpha} \left(L_{i,\alpha} \rho L_{i,\alpha}^\dagger - \frac{1}{2} \{ L_{i,\alpha}^\dagger L_{i,\alpha}, \rho \} \right)$$

[1] Sangchul, O., Lee, S., Lee, H.: Fidelity of quantum teleportation through noisy channels. Phys. Rev.A **66(2)**, 022316 (2002)

[2] Lindblad, G.: On the generators of quantum dynamical semigroups. Commun. Math. Phys. **48(2)**, 119–130 (1976)

[3] Lidar, D.A., Chuang, I.L., Whaley, K.B.: Decoherence-free subspaces for quantum computation. Phys.Rev. Lett. **81(12)**, 2594 (1998)

Noise Analysis

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_s, \rho] + \sum_{i,\alpha} \left(L_{i,\alpha} \rho L_{i,\alpha}^\dagger - \frac{1}{2} \{ L_{i,\alpha}^\dagger L_{i,\alpha}, \rho \} \right)$$

$L_{i,\alpha} = \sqrt{\kappa_{i,\alpha}(t)} \sigma_\alpha^{(i)}$ $\longrightarrow$ Lindblad operator acts on the *ith* qubits and describes the decoherence

$\alpha = X, Y, Z$ $\longrightarrow$ stands for Pauli operations

$\kappa_{i,\alpha}$ $\longrightarrow$ the strength of the decoherence rates $1/\kappa_{i,\alpha}$ $\longrightarrow$ the decoherence time (approximately)

$$H_s(t) = -\frac{1}{2} \sum_{i=1}^N \mathbf{B}^{(i)} \cdot \boldsymbol{\sigma}^{(i)} - \sum_{i \neq j} J_{ij}(t) \sigma_+^{(i)} \sigma_-^{(j)} \longrightarrow \text{Hamiltonian of a qubit system}$$

Noise Analysis

$$H_s(t) = -\frac{1}{2} \sum_{i=1}^N \mathbf{B}^{(i)} \cdot \boldsymbol{\sigma}^{(i)} - \sum_{i \neq j} J_{ij}(t) \sigma_+^{(i)} \sigma_-^{(j)}$$

Where $\boldsymbol{\sigma}^{(i)} = (\sigma_x^{(i)}, \sigma_y^{(i)}, \sigma_z^{(i)})$ & $\sigma_{\pm}^{(i)} = \frac{1}{2}(\sigma_x^{(i)} \pm i\sigma_y^{(i)})$

In solid-state qubits, various types of couplings between qubits *i* and *j* are possible such as *XY coupling* given here

The various quantum gates in our quantum teleportation system which exists in *N3*, *N4* and *N5* noisy channel could be implemented by a sequence of pulses, i.e., by turning on and off $B^{(i)}(t)$ and $J_{ij}(t)$ [1],[2]

[1] Sangchul, O., Lee, S., Lee, H.: Fidelity of quantum teleportation through noisy channels. Phys. Rev.A **66(2)**, 022316 (2002)

[2] Makhlin, Y., Schön, G., Shnirman, A.: Quantum-state engineering with Josephson junction devices. Rev. Mod. Phys. **73**, 357 (2001)

Noise Analysis

It is very useful to describe our quantum teleportation system in terms of density operators.

Hence, we can describe our teleportation system as follows:

$$\rho_{\text{out}} = \text{Tr}_{(a_{n+1} \dots a_{2n}) \text{ or } (b_{n+1} \dots b_{2n})} [U_{\text{tel}} (\rho_{\text{in}} \otimes \rho_{\text{en}}) U_{\text{tel}}^\dagger]$$

where

$$\rho_{\text{in}} = |\Phi\rangle_{A_1 \dots A_n} \langle \Phi| \otimes |\Phi\rangle_{B_1 \dots B_n} \langle \Phi|$$

$$\rho_{\text{en}} = |G\rangle_{a_1 \dots a_n b_1 \dots b_n b_{n+1} \dots b_{2n} a_{n+1} \dots a_{2n}} \langle G|$$

$$\text{Tr}_{(a_{n+1} \dots a_{2n})} \longrightarrow \text{partial trace over every qubits except } (a_{n+1} \dots a_{2n})$$

$$U_{\text{tel}} \longrightarrow \text{The unitary operator which is implemented by our teleportation circuit}$$

Noise Analysis

So, for investigating the noise effect on each channel $N1$ or $N2$ with density matrix ρ , we analytically solve Lindblad equation for ρ and obtain the noisy density matrix $\varepsilon(\rho)$ which is a function of the state of our considered channel, κ , t

our noisy teleported qubits can be calculated as $\varepsilon(\rho_{out})$

we can quantify the properties of our quantum teleportation through noisy channels by computing fidelity between ρ and $\varepsilon(\rho_{out})$ which measures the overlap between these two states as follows:

$$\text{Fidelity} = \langle \Phi |_{A_1 \dots A_n (or B_1 \dots B_n)} \varepsilon(\rho_{out}) | \Phi \rangle_{A_1 \dots A_n (or B_1 \dots B_n)}$$

$\kappa t = 0$  noise has not affected the system  Fidelity=1

Noise Analysis

In order to measure the noise on input ($N1$) and teleportation channel qubits ($N2$), we should consider the value of Hamiltonian for $t = 0$. In this case, the value of H_s is zero

$n=1$  An arbitrary state can be represented as a vector in a Bloch sphere:

$$|\Phi\rangle = \cos\left(\frac{\theta}{2}\right) e^{\frac{i\varphi}{2}} |0\rangle + \sin\left(\frac{\theta}{2}\right) e^{\frac{-i\varphi}{2}} |1\rangle$$

where ϑ is the polar and φ is the azimuthal angle

The density matrix of the input qubits and that of the teleportation channel qubits are defined, respectively, as follows:

$$\begin{aligned}\rho_{\text{in}} &= |\Phi\rangle_{A_1} \langle\Phi| \otimes |\Phi\rangle_{B_1} \langle\Phi| \\ \rho_{\text{en}} &= |G\rangle_{a_1 b_1 b_2 a_2} \langle G|\end{aligned}$$

Noise Analysis

By solving the Lindblad equation for the noise of type Z and X and for ρ_{in} , and ρ_{en} , the values of $\varepsilon(\rho_{in})$, and $\varepsilon(\rho_{en})$ are, respectively, calculated.

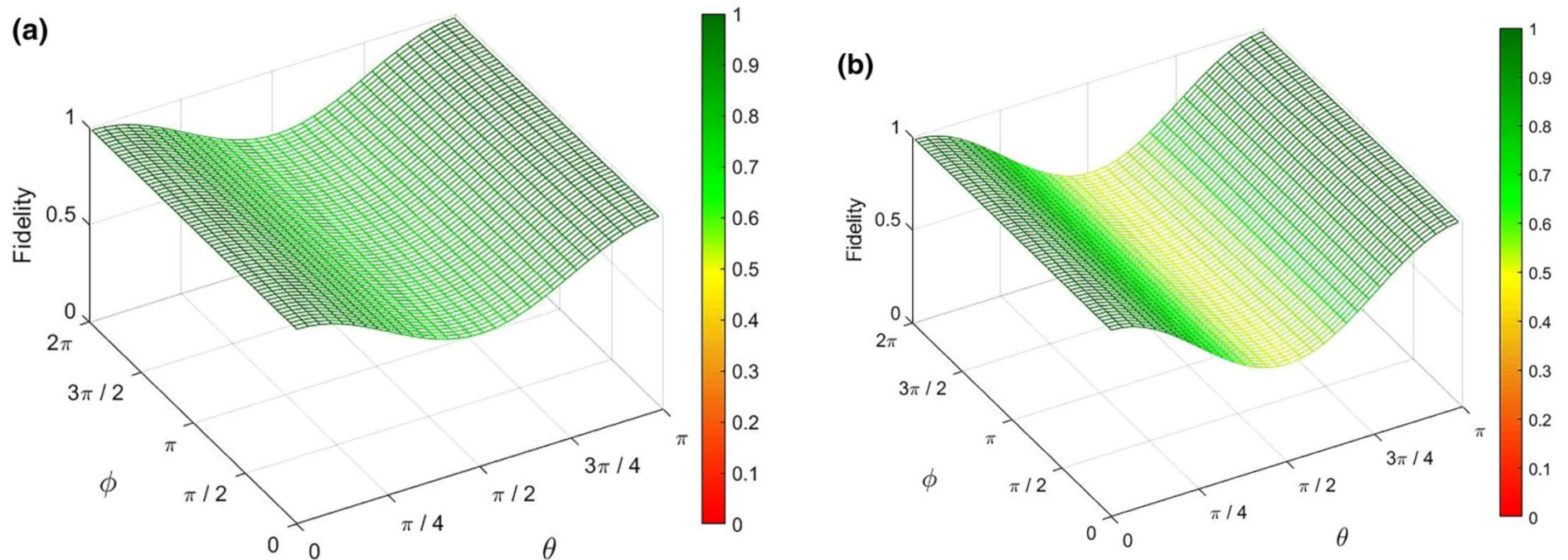
Additionally, for case $n = 1$, the average of the fidelity can be calculated as follows:

$$\text{Fidelity}_{\text{av}} = \frac{1}{4\pi} \int_0^\pi d\theta \int_0^{2\pi} d\phi \text{ Fidelity} \times \sin(\theta)$$

Noise Analysis

Fidelity of Z noise on input qubits (N_1) is as follows:

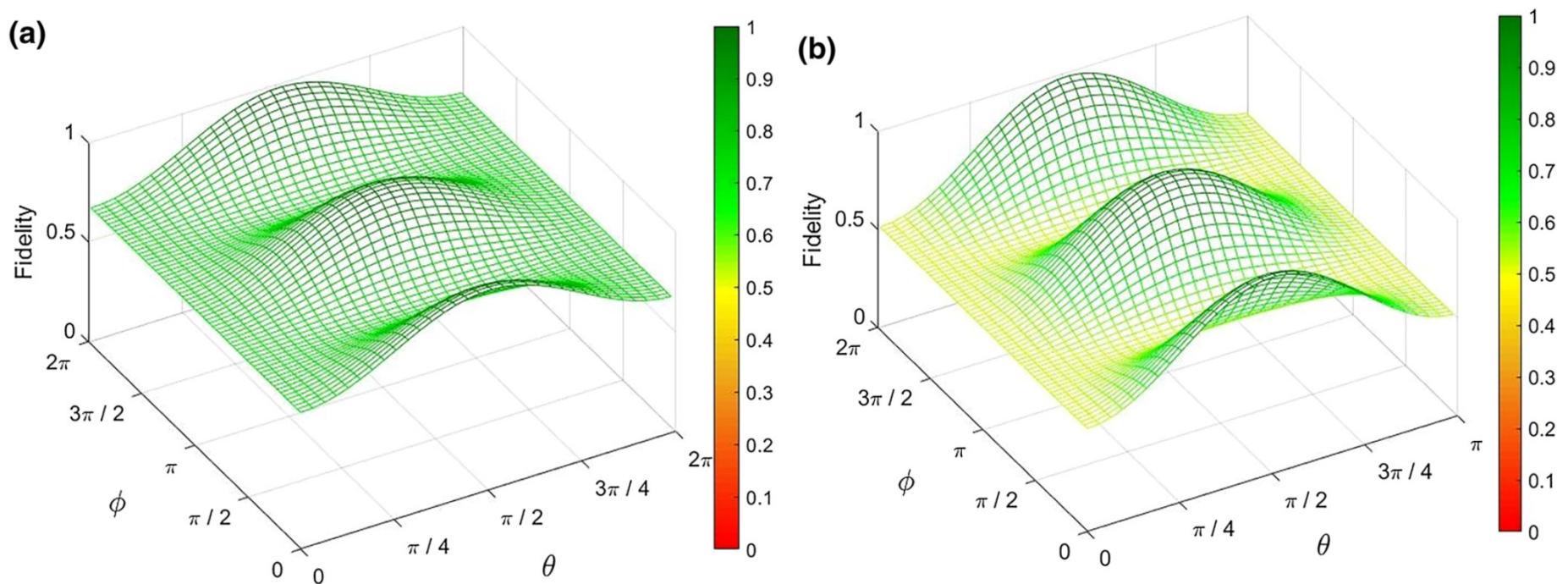
$$\text{Fidelity}(\theta_{A_1}, \phi_{A_1}, \kappa, t) = \frac{1}{4} \left[3 + \cos(2\theta_{A_1}) + 2e^{-2\kappa t} \sin^2(\theta_{A_1}) \right]$$



Fidelity of Z noise. Minimum of **a** is 0.68, and minimum of **b** is 0.52. Maximum for both case is 1. On input qubits (N_1): **a** $\kappa t = 0.5$, **b** $\kappa t = 1.5$. On channel qubits (N_2): **a** $\kappa t = 0.25$, **b** $\kappa t = 0.75$

Noise Analysis

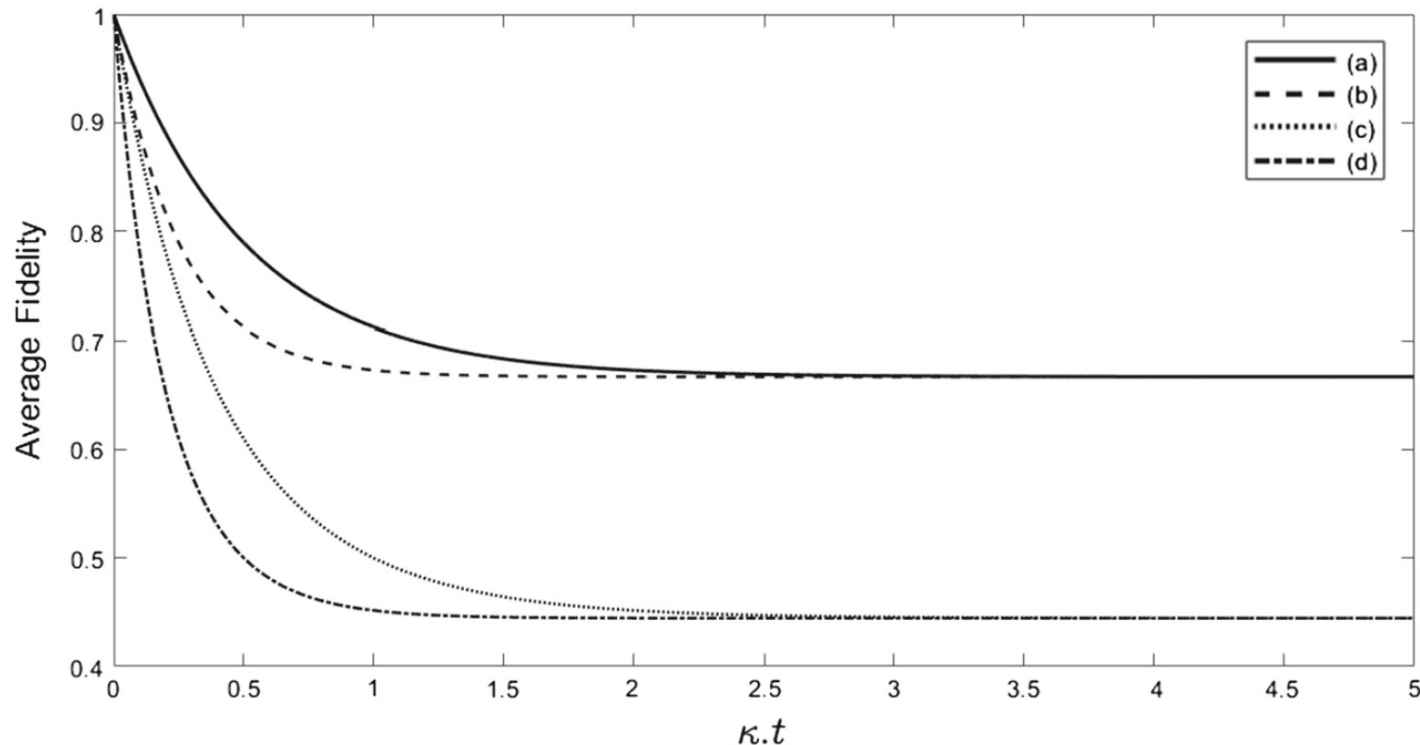
Fidelity of X noise on input qubits ($N1$) is as follows:



Fidelity of X noise. Minimum of **a** is 0.68 and minimum of **b** is 0.52. Maximum for both cases is 1. On input qubits ($N1$): **a** $kt = 0.5$, **b** $kt = 1.5$. On channel qubits ($N2$): **a** $kt = 0.25$, **b** $kt = 0.75$

Noise Analysis

As we previously explained, the average fidelity for *Z noise* and *X noise* are equal [1].



Average fidelity. a $n = 1$, for input qubits (N1). b $n = 1$, for channel qubits (N2). c $n = 2$, for input qubits (N1). d $n = 2$, for channel qubits (N2)

[1] Sadeghi-Zadeh, M.S., Houshmand, M., Aghababa, H. et al. Quantum Inf Process (2019) 18: 353.
<https://doi.org/10.1007/s11128-019-2456-6>

Comparison

References	Year	Protocol type	Number of teleported qubits between Alice and Bob	Quantum channel
[19]	2013	BCQT	1/1	5-qubit composite
[20]	2014	BQT	1/1	4-qubit cluster
[24]	2015	BCQT	1/1	6-qubit genuine
[21]	2015	BCQT	1/1	GHZ type
[13]	2015	BCQT	1/1	8-qubit max Ent.
[22]	2015	BCQT	1/1	6 qubit (3 Bells)
[35]	2016	BQT	1/1	8 qubits
[40]	2016	BCQT	1/2	7 qubits
[41]	2016	BCQT	1/2	7 qubits
[23]	2016	BQT	2(EPR)/2(EPR)	6 qubits (2 GHZs)
[25]	2016	BCQT	2/2	9-qubit cluster
[26]	2016	BCQT	2/1	6-qubit cluster
[27]	2017	BQT	2/2	8 qubits
Proposed protocol	2017	BQT	n/n	$4n$ qubits

Our related works

- [1] Sadeghi-Zadeh, M.S., Houshmand, M., Aghababa, H. et al. , “**Bidirectional quantum teleportation of an arbitrary number of qubits over noisy channel,**” *Quantum Inf Process* 18: 353. <https://doi.org/10.1007/s11128-019-2456-6> , 2019.
- [2] M. Sadeghizadeh, M. Houshmand, H. Aghababa, “**Bidirectional quantum teleportation of a class of n -qubit States by using $(2n+2)$ -qubit,**” *International Journal of Theoretical Physics*, DOI: 10.1007/s10773-017-3551-z, 2018.
- [3] M. Sadeghizadeh, M. Houshmand, H. Aghababa, “**Bidirectional teleportation of a two-qubit State by using eight-qubit entangled state as a quantum channel,**” *International Journal of Theoretical Physics*, DOI: 10.1007/s10773-017-3353-3, 2017.

